Bayesian and Akaike information criteria of the EC-decay rate oscillations *

N. Winckler[1] *and the Two-Body Weak Decay collaboration*[1]
[1]GSI, Darmstadt

Introduction

This report is a continuation of the investigations performed in Ref. [1]. In the previous report we pointed out that the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) could be summarized as follow :

$$\mathrm{IC}_i = -2\log(L(\hat{\theta}_i|\mathrm{data}, M_i)) + A_n K_i \quad (1)$$

and that, in the frequentist interpretation, the decision making in choosing a model M_1 instead of a model M_0 was equivalent to a likelihood ratio test (LRT) in rejecting the null when:

$$-2\log(L_0/L_1) > A_n(K_1 - K_0), \quad (2)$$

where the right-hand term of eq. 1 defines the critical points and where $A_n = 2$ for AIC and $A_n = \log N$ for BIC. From these critical points and the likelihood ratio distribution we could evaluate the AIC- and BIC Type I error rate. Because of a lack of statistical consitency in the ω estimator, minimization procedures were not optimal. Therefore, the resulting maximum likelihood estimates of ω and ϕ were in fact local maxima, distributed arround their initialisation values. As a consequence, the interpretation of the previous calculated LRT distribution involves strong underlying prior probabilities in the $\omega - \phi$ space, restricting values to small parameter intervals. We note that, despite these constraints, the previous computed AIC Type I error rate was already very large ($\approx 20\%$).

Another method bypassing local maxima has been used to obtain the LRT distribution [2]. It turned out that the obtained distribution is much more shifted to higher values, which results in a dramatic increase of the AIC- and BIC Type I error rate.

AIC- & BIC Type I error

The distribution found in Ref. [2] is shown in Figure 1 with the AIC- and BIC critical points. The obtained Type I error rates are shown in Table 1. The fact that the distribution is shifted to higher value is a proof in itself that the global maximum likelihoods have been better estimated. This can be better observed when looking at the distributions of the corresponding Maximum Likelihood Estimates (MLE). The MLE distribution of the angular frequency and phase are now flat under the null hypothesis, as expected for unidentifiable parameters.

* This work is part of the EMMI Rapid Reaction Task Force presentation given in Jena, July 2014

	EC data (245 MHz res.)	β^+ data (245 MHz res.)	EC data (cap. pick-up)
Sample size N	3616	2912	2989
AIC type I error rate	99.3%	99.3%	99.3%
BIC type I error rate	0.07%	0.11%	0.11%

Table 1: AIC- and BIC Type I error rate for different sample sizes

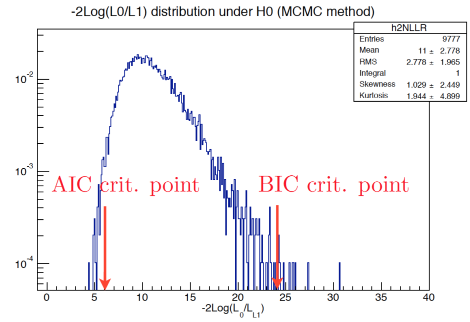

Figure 1: Unbinned maximum likelihood ratio distribution (blue) with the AIC- and BIC critical points (red).

Discussion

It is important to note that, for AIC, these results only depend on the model and on the LRT distribution and do not depend on the data at all. While a probability of 20% to reject the pure exponential decay hypothesis, when it is actually true, can – in this particular problem – cast serious doubts in the reliability of AIC [1], a probability of 99% is obviously not acceptable. Conversely, the BIC has, for the sample size given in table, a much more reasonable Type I error rate of about 0.1%. Accordingly, the BIC is more reliable than AIC in the atempt to reject the null hypothesis of a pure exponential decay. These results have strong consequences in the interpretation of the AIC analysis performed in Ref.[3]. An alternative approach, the computation of the Bayes factor from unbinned likelihood has been performed in Ref. [4].

References

[1] N. Winckler et al., "Further insight into Bayesian and Akaike information criteria of the EC-decay rate oscillations", GSI report (2013)

[2] N. Winckler "Maximum Likelihood Ratio distribution of a modulated exponential decay", GSI report (2014)

[3] P. Kienle et al., "High-resolution measurement of the time-modulated orbital electron capture and of the β^+ decay of hydrogen-like $^{142}\mathrm{Pm}^{60+}$ ions", PLB 726 (2013) 638

[4] N. Winckler et al., "Bayesian Model selection analysis of the EC-decay rate oscillations", GSI report (2014)

DOI:10.15120/GR-2015-1-APPA-MML-AP-14